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Analysis of the spin structure function $g_2(x, Q^2)$ and twist-3 operators *

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Abstract

We discuss the spin-dependent structure function $g_2(x, Q^2)$ in the framework of the operator product expansion. It is noted that the anomalous dimensions and coefficient functions for the twist-3 gluon-field-dependent operators depend on the choice of the operator basis. The role played by the operators proportional to the equation of motion is clarified.

1 Introduction

The spin structure of nucleon is described by the two spin-dependent structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$. Recent experiments at CERN [1, 2] and SLAC [3] have provided new data of $g_1(x, Q^2)$ spin structure function and these data have prompted many authors to reanalyze $g_1(x, Q^2)$ in the connection with the Bjorken sum rule [4]. The measurement of $g_2(x, Q^2)$ was also proposed at CERN and SLAC. The first experimental data was published by the SMC group at CERN [5]. Now theoretical investigations on $g_2(x, Q^2)$ structure function become more and more important.

For $g_1(x, Q^2)$ spin-dependent structure function as well as the spin-independent structure functions $F_1(x, Q^2)$ and $F_2(x, Q^2)$, only twist-2 operators contribute in the leading order of $1/Q^2$ expansion [6]. On the other hand, not only twist-2 operators but also twist-3 operators contribute to $g_2(x, Q^2)$ structure function in the leading order [7, 8].

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In refs. [9, 10], it is pointed out that the operators which are proportional to the *equation of motion* (EOM operator) appear in the twist-3 operators. Due to the presence of the EOM operators, the analysis of $g_2(x, Q^2)$ structure function becomes more complicated than the other structure functions $F_1(x, Q^2)$, $F_2(x, Q^2)$ and $g_1(x, Q^2)$. The appearance of the EOM operators is a general feature in the higher-twist operators.

In this talk, we discuss the structure function $g_2(x, Q^2)$ in the framework of the operator product expansion (OPE) and renormalization group [8]. We focus our attention on the twist-3 operators which contribute to $g_2(x, Q^2)$. Since the twist-3 operators are not all independent, we need to choose the independent operator basis to calculate the anomalous dimensions or the renormalization constants for the twist-3 operators. We study the operator mixing problem with the EOM operators being kept. We clarify the role of EOM operators in the course of the renormalization and point out that the coefficient functions depend upon the choice of the independent operator basis.

2 Definition of $g_2(x, Q^2)$

Let us consider the polarized deep inelastic lepton-nucleon scattering (fig.1).

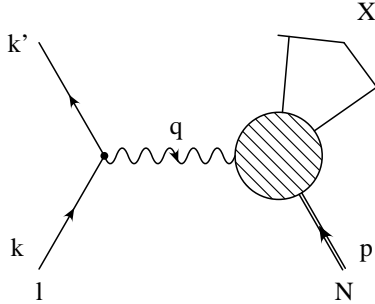


fig.1 $l + N \Rightarrow l + X$

Here we consider only the electro-magnetic interaction between the lepton and nucleon. The kinematical variables p , q , k and k' are defined in fig.1 and $Q^2 = -q^2$. The cross section of this process is given by

$$k'_0 \frac{d\sigma}{d^3k'} = \frac{1}{k \cdot p} \left(\frac{e^2}{4\pi Q^2} \right)^2 L_{\mu\nu} W^{\mu\nu},$$

where $L_{\mu\nu}$ and $W_{\mu\nu}$ are the leptonic and hadronic tensor.

$$\begin{aligned} L_{\mu\nu} &\equiv \frac{1}{2} \langle k | j_\mu(0) | k' \rangle \langle k' | j_\nu(0) | k \rangle \\ W_{\mu\nu} &\equiv \frac{1}{2\pi} \sum_X \langle p | J_\mu(0) | X \rangle \langle X | J_\nu(0) | p \rangle (2\pi)^4 \delta^4(p_X - p - q) \\ &= \frac{1}{2\pi} \int d^4x e^{iq \cdot x} \langle p | [J_\mu(x), J_\nu(0)] | p \rangle. \end{aligned}$$

To define the g_2 structure function, we rewrite the hadronic tensor as follows,

$$W_{\mu\nu} \equiv W_{\mu\nu}^S + iW_{\mu\nu}^A,$$

where $W_{\mu\nu}^S$ is the symmetric part in the Lorentz indices $\mu\nu$ of the hadronic tensor which is described by the two spin independent structure functions $F_1(x, Q^2)$ and $F_2(x, Q^2)$ as,

$$W_{\mu\nu}^S = -\left(g_{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right) F_1 + \left(p_\mu - \frac{p \cdot q}{q^2} q_\mu\right) \left(p_\nu - \frac{p \cdot q}{q^2} q_\nu\right) \frac{2}{\nu M} F_2 \quad .$$

$W_{\mu\nu}^A$ is the antisymmetric part of the hadronic tensor. We can express the antisymmetric part of the hadronic tensor with the two spin dependent structure functions $g_1(x, Q^2)$ and $g_2(x, Q^2)$.

$$W_{\mu\nu}^A = \varepsilon_{\mu\nu\lambda\sigma} q^\lambda \left\{ s^\sigma \frac{g_1}{p \cdot q} + (p \cdot q s^\sigma - q \cdot s p^\sigma) \frac{g_2}{(p \cdot q)^2} \right\}.$$

x is the Bjorken variable given by $x = Q^2/2p \cdot q = Q^2/2M\nu$, $p \cdot q = M\nu$. M is the nucleon mass and s is the covariant spin vector defined by $s^\mu = \bar{u}(p, s) \gamma^\mu \gamma_5 u(p, s)$.

3 Operator product expansion

To apply the OPE to the polarized deep inelastic leptonproduction, we introduce the corresponding forward virtual Compton amplitude $T_{\mu\nu}$ [11].

$$T_{\mu\nu} = i \int d^4x e^{iq \cdot x} \langle p, s | T J_\mu(x) J_\nu(0) | p, s \rangle.$$

From the optical theorem, $T_{\mu\nu}$ and $W_{\mu\nu}$ are related through the relation,

$$W_{\mu\nu} = \frac{1}{\pi} \text{Im} T_{\mu\nu}.$$

The antisymmetric part of the currents product can be expanded with composite operators and Wilson's coefficient functions as,

$$i \int d^4x e^{iq \cdot x} T(J_\mu(x) J_\nu(0))^A = -i \varepsilon_{\mu\nu\lambda\sigma} q^\lambda \sum_{n=1,3,\dots} \left(\frac{2}{Q^2} \right)^n q_{\mu_1} \cdots q_{\mu_{n-1}} \\ \times \left\{ E_q^n R_q^{\sigma\mu_1 \cdots \mu_{n-1}} + \sum_j E_j^n R_j^{\sigma\mu_1 \cdots \mu_{n-1}} \right\}, \quad (1)$$

where R_i^n 's are the composite operators and E_i^n 's are the corresponding coefficient functions. In (1), R_q represents the twist-2 operators and the other operators inside the summation over j are the twist-3 operators. For simplicity, let us consider the flavor non-singlet case. (In the following, we omit the flavor matrices for the quark fields.) The explicit form of the twist-2 operator is given by

$$R_q^{\sigma\mu_1 \cdots \mu_{n-1}} = i^{n-1} \bar{\psi} \gamma_5 \gamma^{\{\sigma} D^{\mu_1} \cdots D^{\mu_{n-1}} \} \psi - (\text{traces}),$$

where $\{ \}$ means symmetrization over the Lorentz indices and $-(\text{traces})$ stands for the subtraction of the trace terms to make the operators traceless. (The trace terms will be omitted in the following.) For the twist-3 operators, we have

$$R_F^{\sigma\mu_1 \cdots \mu_{n-1}} = \frac{i^{n-1}}{n} \left[(n-1) \bar{\psi} \gamma_5 \gamma^\sigma D^{\{\mu_1} \cdots D^{\mu_{n-1}} \} \psi \right. \\ \left. - \sum_{l=1}^{n-1} \bar{\psi} \gamma_5 \gamma^{\mu_l} D^{\{\sigma} D^{\mu_1} \cdots D^{\mu_{l-1}} D^{\mu_{l+1}} \cdots D^{\mu_{n-1}} \} \psi \right], \quad (2)$$

$$R_m^{\sigma\mu_1 \cdots \mu_{n-1}} = i^{n-2} m \bar{\psi} \gamma_5 \gamma^\sigma D^{\{\mu_1} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} \} \psi, \quad (3)$$

$$R_k^{\sigma\mu_1 \cdots \mu_{n-1}} = \frac{1}{2n} (V_k - V_{n-1-k} + U_k + U_{n-1-k}), \quad (4)$$

where m in (3) represents the quark mass (matrix). The operators in (4) contain the gluon field strength $G_{\mu\nu}$ and its dual tensor $\tilde{G}_{\mu\nu} = \frac{1}{2} \varepsilon_{\mu\nu\alpha\beta} G^{\alpha\beta}$ explicitly and are given by

$$V_k = i^n g S \bar{\psi} \gamma_5 D^{\mu_1} \cdots G^{\sigma\mu_k} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} \psi, \\ U_k = i^{n-3} g S \bar{\psi} D^{\mu_1} \cdots \tilde{G}^{\sigma\mu_k} \cdots D^{\mu_{n-2}} \gamma^{\mu_{n-1}} \psi,$$

where S means symmetrization over μ_i and g is the QCD coupling constant.

It is well-known that these operators (2)-(4) are not independent and related through EOM operator .

$$R_{eq}^{\sigma\mu_1\cdots\mu_{n-1}} = i^{n-2}\frac{n-1}{2n}S[\bar{\psi}\gamma_5\gamma^\sigma D^{\mu_1}\cdots D^{\mu_{n-2}}\gamma^{\mu_{n-1}}(i\not{D}-m)\psi \\ +\bar{\psi}(i\not{D}-m)\gamma_5\gamma^\sigma D^{\mu_1}\cdots D^{\mu_{n-2}}\gamma^{\mu_{n-1}}\psi].$$

Making use of the identities $D_\mu = \frac{1}{2}\{\gamma_\mu, \not{D}\}$ and $[D_\mu, D_\nu] = gG_{\mu\nu}$, we can obtain the following relation for the twist-3 operators,

$$R_F^{\sigma\mu_1\cdots\mu_{n-1}} = \frac{n-1}{n}R_m^{\sigma\mu_1\cdots\mu_{n-1}} + \sum_{k=1}^{n-2}(n-1-k)R_k^{\sigma\mu_1\cdots\mu_{n-1}} + R_{eq}^{\sigma\mu_1\cdots\mu_{n-1}}. \quad (5)$$

4 Operator mixing problem

Now we investigate the Q^2 evolution by using the renormalization group method. The presence of the EOM operators brings about some complication through the course of renormalization.

Although the physical matrix elements of the EOM operators vanish [12, 13], we must keep them to study the renormalization because their off-shell Green's function do not vanish. We analyze the operator mixing problem by keeping the EOM operators. In ref.[14] we examined what happens to the renormalization if there are several operators and these operators are related by constraints.

Here we show the one-loop results for the $n = 3$ case as the simplest example. In this case we have four operators with the constraint,

$$R_F = \frac{2}{3}R_m + R_1 + R_{eq}, \quad (6)$$

where the Lorentz indices of operators are omitted.

First we choose the operators $R_F, R_m,$ and R_1 as independent operators and eliminate EOM operator R_{eq} . We get the following renormalization matrix for the composite oper-

ators,

$$\begin{pmatrix} R_F \\ R_1 \\ R_m \\ R_{eq1} \end{pmatrix}_R = \begin{pmatrix} Z_{11} & Z_{12} & Z_{13} & Z_{14} \\ Z_{21} & Z_{22} & Z_{23} & Z_{24} \\ 0 & 0 & Z_{33} & 0 \\ 0 & 0 & 0 & Z_{44} \end{pmatrix} \begin{pmatrix} R_F \\ R_1 \\ R_m \\ R_{eq1} \end{pmatrix}_B \quad (7)$$

where Z_{ij} are given in the dimensional regularization $D = 4 - 2\varepsilon$:

$$Z_{ij} \equiv \delta_{ij} + \frac{1}{\varepsilon} \frac{g^2}{16\pi^2} z_{ij}$$

A straightforward but tedious calculation gives as,

$$\begin{aligned} z_{11} &= \frac{7}{6}C_2(R) + \frac{3}{8}C_2(G), & z_{12} &= -\frac{3}{2}C_2(R) + \frac{21}{8}C_2(G), \\ z_{13} &= 3C_2(R) - \frac{1}{4}C_2(G), & z_{14} &= -\frac{3}{8}C_2(G), \\ z_{21} &= \frac{1}{6}C_2(R) - \frac{1}{8}C_2(G), & z_{22} &= -\frac{1}{2}C_2(R) + \frac{25}{8}C_2(G), \\ z_{23} &= -\frac{1}{3}C_2(R) + \frac{1}{12}C_2(G), & z_{24} &= \frac{1}{8}C_2(G), \\ z_{33} &= 6C_2(R), & z_{44} &= 0. \end{aligned} \quad (8)$$

The quadratic Casimir operators are $C_2(R) = 4/3$ and $C_2(G) = 3$ for the case of QCD.

R_{eq1} is a gauge non-invariant EOM operator.

$$R_{eq1}^{\sigma\mu_1\mu_2} = i\frac{1}{3}S[\bar{\psi}\gamma_5\gamma^\sigma\partial^{\mu_1}\gamma^{\mu_2}(i\not{D} - m)\psi + \bar{\psi}(i\not{D} - m)\gamma_5\gamma^\sigma\partial^{\mu_1}\gamma^{\mu_2}\psi].$$

Although this operator is gauge non-invariant, it is possible to appear in the operator basis because it vanishes by the equation of motion [13, 14]. This result satisfies the equalities.

$$\begin{aligned} z_{11} + z_{12} &= z_{21} + z_{22}, & \frac{2}{3}z_{11} + z_{13} &= \frac{2}{3}z_{21} + z_{23} + \frac{2}{3}z_{33}, \\ z_{13} - \frac{2}{3}z_{12} &= z_{23} - \frac{2}{3}z_{22} + \frac{2}{3}z_{33}. \end{aligned} \quad (9)$$

What happens if we choose R_1 , R_m , R_{eq} and R_{eq1} , and eliminate R_F ? This choice is the same as one adopted by the authors in ref.[9]. Using (6) and relations (9) we get the renormalization matrix

$$\begin{pmatrix} R_1 \\ R_m \\ R_{eq} \\ R_{eq1} \end{pmatrix}_R = \begin{pmatrix} Z_{21} + Z_{22} & \frac{2}{3}Z_{21} + Z_{23} & Z_{21} & Z_{24} \\ 0 & Z_{33} & 0 & 0 \\ 0 & 0 & Z_{11} - Z_{21} & Z_{14} - Z_{24} \\ 0 & 0 & 0 & Z_{44} \end{pmatrix} \begin{pmatrix} R_1 \\ R_m \\ R_{eq} \\ R_{eq1} \end{pmatrix}_B, \quad (10)$$

where Z_{ij} are defined in (7) and (8). In this basis, our results for $n = 3$ case agree with those in ref.[9].

It is to be noted that the renormalization matrix for the operators including EOM operators becomes triangular because the counter-term to the operator R_{eq} should vanish by the equation of motion[13]. Our results are consistent with this general argument.

5 Coefficient function

Next we determine the coefficient functions at the tree level. We used the technique, discussed by E.V. Shuryak and A.I. Vainshtein [9] and R. L. Jaffe and M. Soldate [15]. The coefficient function can be obtained by the short distance expansion of the current products in the presence of the external gauge field.

We include the fermion bilinear operators R_F in the operator basis. We have the coefficient functions,

$$E_q^n(tree) = E_F^n(tree) = 1, E_m^n(tree) = E_k^n(tree) = 0. \quad (11)$$

On the other hand, if we eliminate R_F in the basis, and we use the constraints of the twist-3 operators (6), we have,

$$E_q^n(tree) = 1, \quad E_m^n(tree) = \frac{n-1}{n}, \quad E_k^n(tree) = n-1-k. \quad (12)$$

Now let us see the moment sum rules. General form of the moment for g_2 spin structure function is given by,

$$M_n \equiv \int_0^1 dx x^{n-1} g_2(x, Q^2) = -\frac{n-1}{2n} [a_n E_q^n(Q^2) - d_n E_F^n(Q^2)] + \frac{1}{2} [e_n E_m^n(Q^2) + \sum_{k=1}^{n-2} f_n^k E_k^n(Q^2)]. \quad (13)$$

Here a_n, d_n, e_n and f_n^k are matrix elements of the operators sandwiched between nucleon states with momentum p and spin s ,

$$\langle p, s | R_q^{\sigma\mu_1 \dots \mu_{n-1}} | p, s \rangle = -a_n s^{\{\sigma} p^{\mu_1} \dots p^{\mu_{n-1}\}} \quad (14)$$

$$\langle p, s | R_F^{\sigma\mu_1 \dots \mu_{n-1}} | p, s \rangle = -\frac{n-1}{n} d_n (s^\sigma p^{\mu_1} - s^{\mu_1} p^\sigma) p^{\mu_2} \dots p^{\mu_{n-1}} \quad (15)$$

$$\langle p, s | R_m^{\sigma\mu_1 \dots \mu_{n-1}} | p, s \rangle = -e_n (s^\sigma p^{\mu_1} - s^{\mu_1} p^\sigma) p^{\mu_2} \dots p^{\mu_{n-1}} \quad (16)$$

$$\langle p, s | R_k^{\sigma\mu_1 \dots \mu_{n-1}} | p, s \rangle = -f_n^k (s^\sigma p^{\mu_1} - s^{\mu_1} p^\sigma) p^{\mu_2} \dots p^{\mu_{n-1}} \quad (17)$$

$$\langle p, s | R_{eq}^{\sigma\mu_1 \dots \mu_{n-1}} | p, s \rangle = 0. \quad (18)$$

From the constraint for the twist-3 operators (5), we get relation,

$$\frac{n-1}{n} d_n = \frac{n-1}{n} e_n + \sum_{k=1}^{n-2} (n-1-k) f_n^k. \quad (19)$$

If we choose R_k and R_m as the independent operator basis and use the relation (19), we have the moments for g_2 with the coefficient functions given in (12),

$$M_n = -\frac{n-1}{2n} a_n E_q^n(Q^2) + \frac{1}{2} \left[e_n E_m^n(Q^2) + \sum_{k=1}^{n-2} f_n^k E_k^n(Q^2) \right]. \quad (20)$$

We next check this result by looking at the explicit form of the both side at order of g^2 . Using the perturbative solution of the renormalization group equation, the right-hand side of (20) becomes for the quark matrix elements in the leading order of $\ln Q^2$,

$$\text{RHS of (20)} = -\frac{1}{2} \frac{g^2}{16\pi^2} \left(\frac{n-1}{n} \left(-\frac{1}{2} \right) \gamma_q^0 E_q^n(\text{tree}) + \frac{1}{2} \gamma_{mk}^0 E_k^n(\text{tree}) \right) \ln Q^2 + \dots \quad (21)$$

On the other hand, we can get the $\ln Q^2$ dependence by calculating the one-loop Compton amplitude off the on-shell massive quark target. In the leading order of $\ln Q^2$, the moments becomes,

$$M_n = \frac{1}{2} \frac{g^2}{16\pi^2} C_2(R) \left(-2 + \frac{4}{n+1} \right) \ln Q^2 + \dots \quad (22)$$

From the off-diagonal element of the renormalization constant matrix Z_{km} , the anomalous dimension γ_{mk} reads,

$$\gamma_{mk} \equiv -\frac{g^2}{16\pi^2} \frac{8C_2(R)}{n} \frac{1}{k(k+1)(k+2)} \equiv \frac{g^2}{16\pi^2} \gamma_{mk}^0. \quad (23)$$

This result is in disagreement with the one given in the fifth reference in [9]. The expression for the anomalous dimensions for the twist-2 operators R_F and the quark mass dependent twist-3 operators R_m are given by,

$$\gamma_q^0 = 2C_2(R) \left[1 - \frac{2}{n(n+1)} + 4 \sum_{k=2}^n \frac{1}{k} \right], \quad \gamma_{mm}^0 = 8C_2(R) \sum_{k=1}^{n-1} \frac{1}{k}. \quad (24)$$

Putting (23) and (24) into the above equation with the tree level coefficient functions (12), we find (21) coincides with (22). Thus we confirm that our results(12) and (23) are consistent with the explicit calculation.

6 Summary

We investigate the $g_2(x, Q^2)$ spin structure function in the framework of the OPE. For $g_2(x, Q^2)$, the twist-3 operators contribute in the leading order of the $1/Q^2$ expansion. The twist-3 operators are not independent but constrained through the EOM operators which vanish on application of the equation of motion. Here we demonstrate an analysis for the mixing of the twist-3 operators by keeping the EOM operators. We also determine the coefficient function at the tree level. We pointed out that the coefficient functions depend upon the choice of the independent operator basis.

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